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Confined Phase of QED

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We study resonant pair production in a strong electric field using the complex-time multiple reflection technique. The result is used to explain the occurrence of hadronic-like states of e^+e^- and $\gamma\gamma$ observed in the collision of heavy-ion nuclei. It is suggested that the unusual em background created by the heavy-ion nuclei is equivalent to a gravitational background. We also explain the experimental low-lying bound states, using our approach.

1. INTRODUCTION

Particle creation in a time-dependent electromagnetic field has been an active area of investigation from the standpoint of intense field QED and cosmology (Schwinger, 1961; Parker, 1969; Hartle and Hawking, 1976; Lapedes, 1978; Duru and Unal, 1986; Lotze, 1985, 1989, Barut and Duru, 1989). Recently interest in pair production in time-dependent strong electric fields has grown due to an exciting experimental result obtained from heavy-ion scattering. It is found in experiments on U^+-Th, Th^+-Th, and Th^+-Ca collisions conducted at the Gesellschaft für Schwerionen Forschung, Darmstadt (Kienle, 1987; Tesertos *et al.*, 1987; Reinhardt *et al.*, 1986) that correlated narrow peak structures are observed in electron and positron spectra. The e^+e^- spectra suggest that decays occur from a bound state at rest in the cm system with masses ~1.6–1.8 MeV. In addition, a $\gamma\gamma$ bound state has also been observed at LBL (Danzamann *et al.*, 1987) at 1062 ± 1 keV. The e^+e^- production in heavy-nucleus collisions, showing multiple resonances in pairs, is characterized by equal scalar momenta (most likely oppositely directed) and narrow widths (<40 keV).

Some characteristic features of the resonance structures are (i) they are produced in the Coulomb field of heavy-ion nuclei with a united nuclear charge of $Z = Z_1 + Z_2 > 137$, (ii) resonance masses are $\sim 3m_e$, where m_e is the mass of the electron, (iii) the bound-state formation occurs within a volume $(1/m_e)^3$, i.e., the Compton volume (the electric field is very strong

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within this volume, having both space and time variations, and is relatively unimportant outside the volume), (iv) the electric field is almost constant within the time $t_R \sim 3/m_e \sim 2\times 10^{-21}$ sec, apart from slight oscillatory behavior, and is totally negligible by $t_\gamma \sim 2.5\times 10^{-19}$ sec, when the heavy ions have scattered away, (v) the bound state exists for a time much less than the annihilation time for the e^+e^- pair, and (vi) the location and width of the lines are independent of $Z = Z_1 + Z_2$. All attempts to give a conventional explanation of the effect in terms of pair conversion from excited nuclear states have led to contradictions with the experimental evidence (Schweppe *et al.*, 1983; Tsertos, 1985; Kienle, 1987; Reinhardt *et al.*, 1986). The current ideas that try to explain the formation of bound states are mainly concentrated in three directions. These are (i) the decay of an elementary particle called an axion (Chodos *et al.*, 1986), (ii) the formation of a confined phase of QED like QCD which subsequently decays into an e^+e^- pair or a $\gamma\gamma$ pair (Celenza *et al.*, 1986, 1987; Caldi and Chodos, 1987; Ng and Kikuchi, 1987; Cea, 1989), and (iii) interference effects among different amplitudes (Cornwall and Tiktopoulos, 1989).

The first possibility has been rejected because no particle like the "axion" has been observed so far in e^+e^- colliders (Davies, 1986). Moreover, a sequence of hadronic-like states of e^+e^- goes against the axion hypothesis.

The second possibility has been partially successful, but no model has been able to demonstrate the existence of a confined phase in QED. Some comments may be in order on the QCD-like confined phase of QED (Celenza *et al.*, 1986, 1987; Caldi and Chodos, 1987; Cea, 1989). It is suggested that the heavy ions in the GSI experiment induce significant background electromagnetic fields and these unusual background field environments may give rise to a new phase of QED. The QCD phase is a confined one, whereas the QED phase is a nonconfined one. To understand this, it is suggested that when the background fields disappear, the new phase becomes a false vacuum and the confining potential is then no longer operative. The new phase (false vacuum) then (Coleman, 1977) decays to the familiar vacuum of perturbative QED. The phase is then no longer bound and instead of annihilating each other as in positronium decay, liberation occurs more rapidly than the e^+e^- annihilation. Though the existence of a confined phase has been established in the $U(1)$ case with a possibility of forming a composite object having a level structure, the question remains regarding the deconfining mechanism. The suggestion is that perhaps there is a chiral phase transition as in QCD. In QCD there is a close connection between a chiral transition and a deconfining one.

One serious objection regarding this type of approach is that whereas the origin of the confining phase is clear in QCD, it is not all clear in QED, at least in the present case. The vacuum polarization of QED will be of no help in deciding the existence and the fate of the QED confined phase (Cornwall and Tiktopoulos, 1989).

In this paper we deal with the particle production mechanism in a time-dependent electromagnetic background. The background is chosen such that it might correspond to the situation in heavy-ion scattering or the electromagnetic background in a cosmological problem. In Section 2 we propose a method of calculating the pair production amplitude using a complex-time multiple reflection technique. The method proposed is an extension of the complex-time multiple reflection technique of Knoll and Schaeffer (1977), not in space, but in time. In this sense our approach is a generalization of the Klein paradox situation, not in space but in time, proposed by Cornwall and Tiktopoulos (1989).

In Section 3 we show that the results obtained in Section 2 can also be derived by considering the motion of an electron in a Robertson-Walker space-time. Here we use Bogolubov transformation techniques that are generally used (Birrel and Davies, 1982) to study the particle production mechanism in curved space. The results of Sections 2 and 3 suggest that the simulation of a curved space-time background may be a quite probable situation in heavy-ion scattering.

In Section 4, we use this idea to develop a model that will explain the formation of e^+e^- and $\gamma\gamma$ bound states in heavy-ion scattering. As the LBL results of Danzamann *et al.* (1987) provide a single citation for the $\gamma\gamma$ bound state and the GSI experiments of Darmstadt have not been confirmed by other groups, the model proposed in Section 4 may be considered to show the existence of a confined phase of QED with the gravitational background $(4-1)$ as an ansatz. Further experiments on heavy-ion scattering will decide whether the confined phase of QED exists or not.

Recently Barut and Duru have considered the pair production in an electric field in a time-dependent gauge with spontaneous pair production and suggested that such a pair production in the em case is more parallel to some models of an expanding universe. We also obtain the same result as Barut and Duru (1989). In a recent communication (Biswas and Kumar, 1990) we have used a potential $eA_3 = eA$ for $0 \le t \le t_A$ and zero otherwise to obtain the result of Cornwall and Tiktopoulos (1989) using the same complex-time reflection technique.

2. COMPLEX-TIME MULTIPLE REFLECTION

Let us start with a one-dimensional Schrödinger equation not in space but in time,

$$\left[\frac{d^2}{dt^2} + w^2 - U(t)\right]\psi(t) = 0 \tag{2.1}$$

Let us assume that $U(t)$ is an analytic function of t and there exist two complex turning points t_1 and t_2 given by

$$w^2 - U(t_1, t_2) = 0 \tag{2.2}$$

The classical paths contributing in a complex semiclassical approximation, joining two prescribed real points t' and t'' (Fig. 1), are those with zero, one, two reflections between two complex turning points t_1 and t_2. The multiple reflection series for $\psi(t'', t')$ depends on the situation of the region of analyticity of the WKB approximants $\exp(-i, S_{1,2})$ with classical action

$$S_{1,2}(t_{1,2}, t) = \int_{t_{1,2}}^{t} dt' \, [w^2 - U(t')]^{1/2} \tag{2.3}$$

in the complex t plane. Let us quote the resulting multiple reflection series when $t' = \infty$ and t'' are on the right of t_1 and t_2. The multiple expansion reads (Knoll and Schaeffer, 1977)

$$\begin{aligned}\psi(t''; \infty) \sim \frac{1}{[w^2 - U(t'')]^{1/4}} \Big(& \exp[-iS(t'', \infty)] \\ & - i \exp\{(-i)[S(t_1, \infty) - S(t'', t_1)]\} \\ & \times \sum_{\mu=0}^{\infty} \{-i \exp[-iS(t_2, t_1)]\}^{\mu} \Big)\end{aligned} \tag{2.4}$$

with

$$\sum_{\mu=0}^{\infty} = 1/\{1 + \exp[-2iS(t_2, t_1)]\} \tag{2.5}$$

where

$$\begin{aligned} S(t'', \infty) &= \lim_{t' \to \infty} \left\{ \int_{t'}^{t''} dt \, [w^2 - U(t)]^{1/2} + wt' \right\} \\ &= \int_{\infty}^{t''} dt \, \{[w^2 - U(t)]^{1/2} - w\} + wt'' \end{aligned} \tag{2.6}$$

with

$$\mathrm{Re}[w^2 - U(t)]^{1/2} > 0$$

The diagrammatic representations of (2.5) are given in Figure 1.

The interpretation of (2.4) is as follows. The classical trajectories building up the quantum mechanical wave are the direct (real) trajectory from $t' = \infty$ to t'' represented by the first term in the expression (2.4) and the trajectory from ∞ returning to t'' after "complex" reflections between t_1 and t_2 leading to the geometrical series (2.4) and (2.5). In the usual WKB

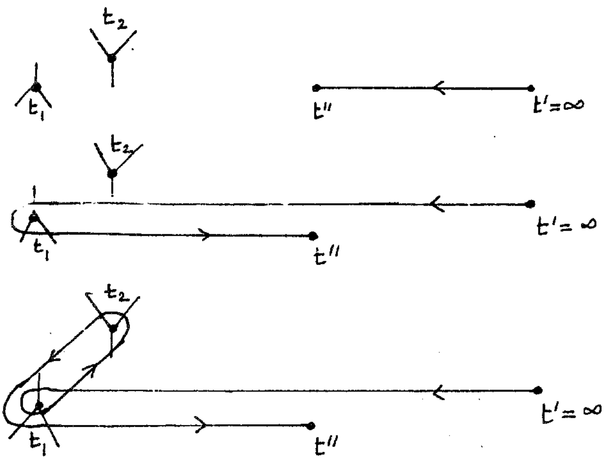

Fig. 1. (a) No reflection, (b) one reflection, and (c) reflections at t_1, t_2, and t_1.

approximation with real semiclassical path we have only the first term $\exp[-iS(t'', \infty)]$ in (2.4). The reflection coefficient for this one-dimensional problem is given by

$$\psi(t'', \infty) \underset{t'\to\infty}{\sim} \exp(iwt'') + iR\exp(-iwt) \tag{2.7}$$

such that

$$R = \frac{\exp[-2iS(\infty, t_1)]}{1+\exp[-2iS(t_2, t_1)]} \tag{2.8}$$

Hence for certain complex values of the multiple reflection terms, (2.8) may exhibit poles if

$$S(t_2, t_1) = (N+\tfrac{1}{2})\pi \tag{2.9}$$

with N integer. Such poles may add up to give nonperturbative contributions enhancing R.

In order to apply the result to the creation of fermion pairs, $\exp(\mp iwt)$ terms in (2.7) must be replaced by free particle solutions of the Dirac equation. We now show that the Dirac equation in a time-dependent electric field can be brought to the form (2.1) leading to an expression like (2.7). We consider the Dirac equation

$$(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - M)\psi(x) = 0 \tag{2.10}$$

We choose the electromagnetic potential to be time dependent,

$$A_{\mu} = (0, 0, 0, Et) \tag{2.11}$$

The more familiar potential

$$A'_\mu = (-Ez, 0, 0, 0) \tag{2.12}$$

is related to A_μ by a gauge transformation

$$A'_\mu = A_\mu + \partial_\mu X \tag{2.13}$$

with

$$X = -Ezt \tag{2.14}$$

Putting (2.12) in (2.10), we get

$$\{i\partial_0 + \alpha_3[P_3 + eA_3(t)] - \beta' m\} X_j(t) = 0 \tag{2.15}$$

after substituting

$$\psi_j = X_j(t) \exp(i\mathbf{P} \cdot \mathbf{x}) \tag{2.16}$$

For $j = 2$, P will correspond to the negative of antiparticle momentum. In (2.15)

$$\alpha_i = \gamma_0 \gamma_i, \qquad \beta' m = \gamma_0 M - \boldsymbol{\alpha}_\perp \cdot \mathbf{P}_\perp, \qquad m = (P_\perp^2 + M^2)^{1/2} \tag{2.17}$$

As equation (2.15) contains two anticommuting variables, we can choose

$$\beta' = \sigma_3, \qquad \alpha_3 = -\sigma_1$$

to write (2.15) as

$$\{-i\partial_0 + \sigma_1[P_3 + eA_3(t)] + \sigma_3 m\} X_j(t) = 0 \tag{2.18}$$

The free particle solutions of (2.18) are

$$X_{10} = \begin{pmatrix} \cos \frac{1}{2}\theta \\ \sin \frac{1}{2}\theta \end{pmatrix} \exp(-iwt) \tag{2.19}$$

$$X_{20} = \begin{pmatrix} -\sin \frac{1}{2}\theta \\ \cos \frac{1}{2}\theta \end{pmatrix} \exp(iwt) \tag{2.20}$$

where

$$\cos\theta = \frac{m}{w}, \qquad \sin\theta = \frac{P_3}{w} \tag{2.21}$$

The second-order Dirac equation corresponding to (2.15) is

$$[\partial_0^2 - w^2 - P_3^2 + (P_3 + eA_3)^2 + i\alpha_3 eE]\psi_j(x) = 0 \tag{2.22}$$

Equation (2.22) is of the form (2.1) with

$$U(t) = P_3^2 - [P_3 + eA_3(t)]^2 \pm ieE \tag{2.23}$$

The $\pm$ sign before eE is due to the α_3 term. We will find that this term causes an enhancement of pair production amplitude compared to scalar particle pair production. The multiple reflection series for the case of a Dirac particle is now written as

$$\psi(t) \sim X_{10}(t) + iRX_{20}(t) \tag{2.24}$$

where X_{10} and X_{20} are given by (2.19) and (2.20). It is assumed that there is a multiplicative factor $\exp(i\mathbf{P}\cdot\mathbf{x})$ in the rhs of (2.24). The turning points are now given by

$$w^2 - U(t) = 0$$

so that

$$\begin{aligned} P_3 + eEt_1 &= -im \\ P_3 + eEt_2 &= +im \end{aligned} \tag{2.25}$$

where

$$m = (P_\perp^2 + M^2 \pm ieE)^{1/2} \tag{2.26}$$

Let us now evaluate $S(t_2, t_1)$ and $S(\infty, t_1)$. From (2.3)

$$S(t_2, t_1) = -\int_{t_2}^{t_1} dt' \{m^2 + [P_3 + eA_3(t')]^2\}^{1/2} \tag{2.27}$$

Let us put

$$P_3 + eEt' = iT$$

in (2.27). We get

$$\begin{aligned} S(t_1, t_2) &= \frac{i}{eE}\int_{-m}^{+m} (m^2 - T^2)^{1/2}\, dT \\ &= \frac{im^2}{eE}\int_{-m}^{+m} \left(1 - \frac{T^2}{m^2}\right)^{1/2} d\left(\frac{T}{m}\right) \end{aligned}$$

Now letting $T/m = \sin\theta$, we get

$$S(t_2, t_1) = \frac{im^2}{2eE}\pi \tag{2.28}$$

Now

$$\begin{aligned} S(\infty, t_1) &= \lim_{\varepsilon\to 0} S(\infty, \varepsilon) + S(0, t_1) \\ &= \lim_{\varepsilon\to 0} (w\varepsilon) + \frac{im^2}{2eE}\frac{\pi}{2} \end{aligned} \tag{2.29}$$

so that the reflection amplitude is

$$R = \frac{\exp[-2iS(\infty, t_1)]}{1+\exp[-2iS(t_2, t_1)]}$$
$$= \frac{\exp(m^2\pi/2eE)}{1+\exp(\pi m^2/eE)} \tag{2.30}$$

The enhancement of R is due to poles at complex time given by

$$\frac{im^2}{2eE}\pi = \left(N+\frac{1}{2}\right)\pi$$

or

$$im^2 = 2eE\left(N+\frac{1}{2}\right)$$

Let us introduce

$$m^2/eE = (P_\perp^2 + M^2 \pm ieE)/eE$$
$$= \mu^2/eE \pm i$$
$$= \lambda \pm i \tag{2.31}$$

For scalar particle pair production, the $\pm i$ term will be absent. In that case, using (2.28) and (2.29), we get

$$|R|^2 = \frac{\exp \pi\lambda}{(1+\exp \pi\lambda)^2} \tag{2.32}$$

For the spin-1/2 case,

$$|R|^2 = \frac{\exp \pi\lambda}{(1-\exp \pi\lambda)^2} \tag{2.33}$$

The result (2.32) is also obtained by Barut and Duru (1989) by a different method; however, (2.33) differs in our case. Equation (2.32) or (2.33) gives the probability for one pair production. The average number of pairs summed over all modes can be easily evaluated by approximating (2.32) and (2.33) as $\exp(-\pi\lambda)$. Thus (Barut and Duru, 1989),

$$\bar{N} = \int dP_x \frac{L}{2\pi} \int dP_y \frac{L}{2\pi} \int_0^{eEL} dP_0 \frac{T}{2\pi} \exp(-\pi\lambda)$$
$$= \frac{(eE)^2 L^3 T \exp(-\pi\mu^2/eE)}{(2\pi)^3} \tag{2.34}$$

which is in agreement with the well-known result.

3. ROBERTSON–WALKER SPACE-TIME AND PARTICLE CREATION

For a time-dependent field the vacuum is very complicated. Actually, one does not know what the vacuum is in the Klein paradox situation, particularly for a strongly varying, time-dependent potential where the production of real pairs occurs or the gas $2m_ec^2$ is close (which is actually the case in heavy-nucleus scattering). To define the vacuum, one needs mode solutions corresponding to the equation of motion. In our case, initially the mode solutions are defined such that

$$\frac{\partial}{\partial t} X_j(t) = -iwX_j(t) \tag{3.1}$$

corresponding to

$$(i\partial_0 + \alpha_3 P_3 - \beta' m)X_j(t) = 0 \tag{3.2}$$

Let the vacuum defined by these mode solutions be $|0_{\rm in}\rangle$, i.e.,

$$\hat{a}_j|0_{\rm in}\rangle = 0 \tag{3.3}$$

where

$$\psi_2(t) = \sum_j [\hat{a}_j X_j(t) + \hat{a}_j^+ X_j^*(t)] \tag{3.4}$$

with $\hat{a}_j^+$ and $\hat{a}_j$ as creation and annihilation operators. In a time-dependent field there is a possibility of mixing of positive- and negative-energy states and the meaning of the definition (3.1) is lost. Actually this is the trouble in the Klein paradox situation. Due to pair creation the positive energy component at $t \to +\infty$ will get contributions from the negative-energy components (due to mixing), resulting in different asymptotic states at $t \to \infty$ and $t \to -\infty$. Moreover, the behavior of time-dependent $A(t)$ at $t \to \pm\infty$ will also be different, at least by a constant term i.e.,

$$|\mathbf{A}(t \to +\infty) - \mathbf{A}(t \to -\infty)| = \text{const}$$

In Minkowski space-time the vector $\partial/\partial t$ is a Killing vector orthogonal to the spacelike hypersurface $t = \text{const}$ and the vacuum is invariant under the action of the Poincaré group. If for some reason the Poincaré symmetry is lost (this situation occurs in curved space-time) one has to define another set of mode solutions x_j such that

$$\psi_2(t) = \sum_j [\bar{\hat{a}}_j \bar{X}_j(t) + \bar{\hat{a}}_j^+ \bar{X}_j^*(t)] \tag{3.5}$$

and a new vacuum and a new Fock space. As both sets are complete, one can write (Birrel and Davies, 1982)

$$\bar{X}_j = \sum_i (\alpha_{ji} X_i + \beta_{ji} X_i^*) \tag{3.6}$$

$$X_i = \sum_j (\alpha_{ji}^* \bar{X}_i + \beta_{ji} \bar{X}_j^*) \tag{3.7}$$

where both sets form a complete set. Whenever the Bogolubov coefficient $\beta_{ij} \neq 0$ there is particle creation and the number of particle created is given by

$$\langle \bar{0} | N_i | \bar{0} \rangle = \sum_j |\beta_{ji}|^2 \tag{3.8}$$

To find β_{ji}, we may start with (2.22), finding "in" and "out" mode solutions. We find $X_{\text{in}} \neq X_{\text{out}}$, showing that $|\beta_{ij}|^2 \neq 0$, i.e., the strong electric field mixes the positive-energy and negative-energy states. As a result, the Poincaré symmetry is lost and the space-time is not fully Minkowskian. In order to understand this statement, we start from a RW space-time to obtain equation (2.22) for a particular choice of radius parameter of RW space-time. The results (2.32) and (2.33) are also obtained from the curved space-time technique mentioned above. Thus, the simultation of a gravitational background by the strong electric field seems convincing.

The Coulomb fields of the heavy-ion nuclei is of the form

$$F(t) = \frac{Ze^2}{[r(t)]^2} \tag{3.9}$$

where $r(t)$ is measured from the point where the two nuclei would have collided in the absence of Coulomb interactions. The time dependence can be evaluated knowing the trajectories of the colliding nuclei. As the behavior of the Dirac electron at very short distance is unusual (Landau and Lifshitz 1982) we take a different course, writing (3.9) as

$$F(t) = \frac{Ze^2}{a^2(t) r^2} \tag{3.10}$$

Equation (3.10) implies a transformation $r^2 \to a^2(t) r^2$, i.e.,

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{3.11}$$

i.e., the possibility of describing the time-dependent Coulomb field of nuclei by RW space-time. In principle one should determine $a(t)$ locally by writing a Lagrangian density of the $a(t)$ field, em field, and electron field. Let us try to establish the equivalence between the description given in Section 2 and (3.11). For simplicity, let us write down the line element (3.11) for the one-dimensional problem. The generalization to three dimensions is straightforward. We write

$$ds^2 = dt^2 - a^2(t)\, dx^2 \tag{3.12}$$

as

$$ds^2 = a(\eta)(d\eta^2 - dx^2) \tag{3.13}$$

where

$$t = \int^t dt' = \int^\eta a(\eta')\, d\eta' \tag{3.14}$$

It should be pointed out that the line element (42) is manifestly conformal to Minkowski space-time. Suppose we start with a scalar field equation

$$(\Box_x + M^2)\psi(x) = 0 \tag{3.15}$$

The mode solutions of (3.15) can be written as (Birrel and Davies, 1982)

$$u_x(\eta, x) = (2\pi)^{-1/2} \exp(ikx)\, X_k(x) \tag{3.16}$$

where X_k now satisfies

$$\frac{d^2}{d\eta^2} X_k(\eta) + [k^2 + a(\eta) M^2] X_k(\eta) = 0 \tag{3.17}$$

If now for a particular choice of $a(\eta)$ the mode solutions

$$X_k^{\text{in}}(\eta) \neq X_k^{\text{out}}(\eta)$$

we say that there is particle creation by a time-dependent gravitational field (in our case it is the em field). Let us choose

$$a(\eta) = a^2 + b^2\eta^2 \tag{3.18}$$

which is the well-known solved example for the spacetime (3.13). The exact solutions of (3.17) and then, for $a(\eta)$ given by (3.18),

$$X_k^{\text{in}}(\eta) = (2Mb)^{1/4} \exp\left(-\frac{\pi\lambda}{8}\right) D_{-(1-i\lambda/2)}[(i-1)(Mb)^{1/2}\eta] \tag{3.19}$$

$$X_k^{\text{out}}(\eta) = X_k^{\text{in}}(-\eta) \qquad \text{for} \quad \eta > 0 \tag{3.20}$$

In the above expression

$$\lambda = \frac{a^2 M}{b} + \frac{k^2}{Mb} \tag{3.21}$$

and D_ν and parabolic cylinder functions. Further,

$$X_k^0(\eta) \xrightarrow[\eta \to \pm\infty]{} (2M|\eta|b)^{-1/2} \exp(\mp iMb\eta^2/2) \tag{3.22}$$

As $X_k^{\text{in}} \neq X_k^{\text{out}}$, we define two adiabatic vacuums $|0_{\text{in}}^A\rangle$ and $|0_{\text{out}}^A\rangle$ that are not identical. This means that the Bogolubov coefficient is $|\beta_{ij}|^2 \neq 0$. All these are standard results. To evaluate the Bogolubov coefficient, we have the relation

$$U_k^{\text{in}} = \frac{i(2\pi)^{1/2}\exp(-\pi\lambda/4)}{\Gamma(\frac{1}{2}(1-i\lambda))} U_k^{\text{out}} - i\exp\left(-\frac{\pi\lambda}{2}\right) U_k^{\text{out}} \tag{3.23}$$

so

$$|\beta_k|^2 = \exp(-\pi\lambda) = \exp\left[-\pi\left(\frac{k^2}{Mb}+\frac{Ma^2}{b}\right)\right] \tag{3.24}$$

To bring our Dirac equation (2.22) to the form (3.17) to find $a(t)$, we put

$$\eta = (t + P_3/eE)$$
$$A_3(t) = eEt$$

to get

$$\left[\frac{d^2}{d\eta^2} + P^2 - P_3^2 + M^2 + ieE + (eE)^2\eta^2\right] X_j(t) = 0 \tag{3.25}$$

and identify

$$K^2 = P^2, \qquad -P_3^2 + M^2 + ieE = a^2, \qquad eE = b \tag{3.26}$$

Thus, the Dirac particle of mass M moving in a field $A_\mu = (0, 0, 0, Et)$ can be viewed as the motion of a particle of unit mass in RW space-time (3.13) with

$$a(\eta) = (ieE + M^2 - P_3^2) + (eE)^2(t + P_3/eE)^2 \tag{3.27}$$

Actually, one should start from the first-order Dirac equation to show the equivalence; however, (3.25) can also be considered as a Dirac equation with ψ as a four-components object.

If one considers a Klein–Gordon particle, the number of particles produced is, from (3.24),

$$|\beta_k|^2 = \exp[-\pi(M^2 + P_\perp^2)/eE]$$
$$= \exp(-\pi\lambda)$$

which is the same result (2.31) that we obtained from the complex-time reflection technique. For fermion pairs the result is again

$$|\beta_k|^2 = |\exp(-\pi\lambda)|^2$$

but

$$\lambda = (P_\perp^2 + M^2 \pm ieE)/eE$$

However, the pole position of the amplitude is now given by [from (3.23)]

$$\tfrac{1}{2}(1-i\lambda)=-N$$

occurring from the poles of the Γ function. Thus, we get

$$i(m^2+ieE)=2eE(N+\tfrac{1}{2}) \tag{3.28}$$

This is exactly the same result that we obtained from the multiple reflection technique in Section 2. These poles in R are very crucial to the understanding of the reflection of the positron (electron) from the vacuum, i.e., the mechanism of pair production (Cornwall and Tiktopoulos, 1989).

4. THE MODEL

In this section we propose a model to show the existence of e^+e^- and $\gamma\gamma$ bound states analogous to quark-antiquark bound states and glueball states of QCD. This type of model was originally considered by Dicke (1957) to deal with gravitation as a sort of electromagnetic effect. We have seen in previous sections that pair production in a time-dependent electromagnetic field can equivalently be described by RW space-time. It is thus assumed that the scale factor of RW space-time becomes time independent when the production ceases and is described by a background given by

$$ds^2=1/\varepsilon(r)\,dt^2-\varepsilon(r)(dx^2+dy^2+dz^2) \tag{4.1}$$

Equation (4.1) is taken as an ansatz for the case when the heavy ions are scattered away. One may argue that the realistic situation in heavy-ion scattering does not correspond to the case (4.1). Our suggestion is that for a model background like (4.1), it is possible to show the existence of a confining potential as well as quark-antiquark-like bound states for e^+e^- and $\gamma\gamma$. With this assumption we take for the Lagrangian density of the model (Dicke, 1957)

$$L=L_f+L_{\text{em}}+L_\psi \tag{4.2}$$

where

$$L_f=\frac{1}{2k}f^{\mu\nu}\,\partial_\nu\varepsilon\,\partial_\mu\varepsilon \tag{4.3}$$

$$L_{\text{em}}=-\frac{\varepsilon}{16\pi}F_{\mu\nu}f^{\mu\alpha}F_{\alpha\beta}f^{\beta\nu} \tag{4.4}$$

$$L_\psi=-\frac{i}{2}\bar{\psi}\tilde{\gamma}^\mu\,\partial_\mu\psi \tag{4.5}$$

For now we take the electron as massless. The mass term will be included in the proper place. In equations (4.3)-(4.5), $f_{\mu\nu}$ are given by (4.1) and

$$\tilde{\gamma}^i = \varepsilon^{-1/2}(r)\nu^i \tag{4.6}$$

$$\tilde{\gamma}^0 = \varepsilon^{+1/2}(r)\gamma^0 \tag{4.7}$$

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \tag{4.8}$$

The forms (4.3)-(4.8) are dictated by a scaling of length and time measurements as (Dicke, 1957)

$$L = L_0\varepsilon^{-1/2} \tag{4.9}$$

$$\omega = \omega_0\varepsilon^{-1/2} \tag{4.10}$$

so that we practically work in a flat Newtonian coordinate system

$$ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu \tag{4.11}$$

with $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. To distinguish the model from the hadronic case, we mention that the coupling constant K of the scalar field ε is a parameter of the theory, which in the present case would be $\sim m_e^2$ instead of m_π^2 in the hadronic case.

The variational principle corresponding to (4.9)-(4.11) is now written as

$$\delta \int L(-\eta)^{1/2}\, d^4x = 0 \tag{4.12}$$

The field equations obtained are

$$\nabla^2\varepsilon - \frac{(\nabla\varepsilon)^2}{2\varepsilon} = -K\left(\frac{\varepsilon E^2}{8\pi} + \frac{B^2}{8\pi\varepsilon} - \omega\bar{\psi}\gamma^0\varepsilon^{1/2}\psi\right) \tag{4.13}$$

$$\tilde{\gamma}^\mu\, \partial_\mu\psi = 0 \tag{4.14}$$

$$\nabla \times (\mathbf{B}/\varepsilon) - (\partial/\partial t)(\varepsilon\mathbf{E}) = 0 \tag{4.15a}$$

$$\nabla \cdot (\varepsilon\mathbf{E}) = 0 \tag{4.15b}$$

In (4.13) we assumed $\psi \sim \exp(-iwt)$, and we used (4.14) to get the rhs of (4.13). To understand the origin of the confined phase, we note that when one calculates the energy density from the expression

$$T^\mu_\nu = \xi_{,\nu}\frac{\partial L_\xi}{\partial\xi_{,\mu}} - \delta^\mu_\nu L_\xi \tag{4.16}$$

one finds that the second term on the lhs of (4.13) is the energy density of the scalar field ε. The first two terms on the rhs correspond to the energy density of the em field and the third term is that for the electron field. In the sense of general relativity we say that the source of ε is the energy density of the field. The signs of these terms indicate that there may be cancellation between them to generate a proper ε for having a confined phase of QED (Dicke, 1957). As these energy terms are dominant at short distances, the occurrence of a confining phase even in QED seems convincing from (4.13). To generate a proper ε, characteristic of a confined phase, we choose the boundary conditions

$$\begin{aligned} \varepsilon(r) &\xrightarrow[r\to 0]{} 1 \\ \varepsilon(r) &\xrightarrow[r\to\infty]{} \text{finite} \end{aligned} \tag{4.17}$$

The exact solutions of (4.13)–(4.15) can be obtained numerically along the lines of Aoki *et al.* (1989). However, we look for an analytical solution approximating the rhs of (4.13) by 2 λ_f and put

$$\varepsilon^{1/2} = \exp(2\lambda)$$

in (4.13) and then

$$F = r\exp(2\lambda)$$

to get

$$F'' + 2\lambda_f r = 0$$

with the solutions

$$F = a + br - \tfrac{1}{3}\lambda_f r^3 \tag{4.18}$$

In view of (4.17), we take $a=0$ and $b=1$ in (4.18) as $F\to r$ as $r\to 0$. The required solution is

$$\varepsilon = (1 - \tfrac{1}{3}\lambda_f r^2)^2 \tag{4.19}$$

for $r \ll R$. The surface characterized by $r = R$ separates the confining phase from the nonconfining one.

Now, to show the existence of bound states for both the electron and the photon we put (4.19) in (4.14) and (4.15). Using the approximation $\lambda_f^{1/2} r \ll 1$ and

$$\psi \sim \exp(-\lambda_f r^2/2 - iwt)\begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{4.20}$$

we find that equation (4.14) reduces to

$$\begin{aligned} \xi' &= \frac{k-1}{r}\,\xi - \omega\varepsilon\eta \\ \eta' &= \omega\varepsilon\xi - \frac{k+1}{r}\,\eta \end{aligned} \tag{4.21}$$

Now looking back at the photon field equations (4.15) and using

$$\mathbf{E}=\partial\mathbf{A}/\partial t-\nabla\phi, \qquad \mathbf{B}=\nabla\times\mathbf{A}, \qquad \varepsilon'=\partial\varepsilon/\partial r \tag{4.22}$$

we convert (4.15) to

$$\nabla^2\mathbf{A}+\frac{\varepsilon'}{\varepsilon r}(\mathbf{r}\times\nabla\times\mathbf{A})+\omega^2\varepsilon^2\mathbf{A}=\nabla(\nabla\cdot\mathbf{A})-i\omega\varepsilon^2\nabla\phi \tag{4.23}$$

To make (4.21) homogeneous and to get cavitylike eigenmodes, we enforce the gauge condition (Biswas and Kumar, 1990; Khadkikar, 1987)

$$\nabla(\nabla\cdot\mathbf{A})-i\omega\varepsilon^2\nabla\varphi=0$$

As $r\to 0$, the Lorentz condition is satisfied. However, as $r\to(3/\lambda_f)^{1/2}$ lies in the confinement region, the condition $\nabla\cdot\mathbf{A}=0$ is well satisfied. Equation (4.22) then reduces to

$$\nabla^2\mathbf{A}-\frac{\varepsilon'}{\varepsilon}\frac{\partial\mathbf{A}}{\partial r}+\omega^2\varepsilon^2\mathbf{A}=0 \tag{4.24}$$

Thus, in the region $\nabla\cdot\mathbf{A}=0$, eigenmodes can be defined as

$$\mathbf{A}^{\mathrm{TE}}=\mathbf{L}\psi_{nlm}$$

$$\mathbf{A}^{\mathrm{TM}}=\nabla\times\mathbf{L}\psi_{nlm}$$

with $\psi_{nlm}=R_{nl}\gamma_{lm}(\theta,\varphi)$ and R_{nl} now satisfies the equation

$$R''_{nl}+\left(\frac{2}{r}-\frac{\varepsilon'}{\varepsilon}\right)R_{nl}+\left(\omega^2\varepsilon^2-\frac{l(l+1)}{r^2}\right)R_{nl}=0 \tag{4.25}$$

The pair (4.21) is now converted into a second-order equation,

$$\xi''+\left(\frac{2}{r}-\frac{\varepsilon'}{\varepsilon}\right)+\left(\omega^2\varepsilon^2-\frac{k(k+1)}{r^2}\right)\xi=0 \tag{4.26}$$

The structure of (4.25) and (4.26) suggests that the electrons and the photons are confined in the same way in the gravitational background. However, we have to include the mass term for the electron case. The solutions of (4.25) or (4.26) have been carried out elsewhere (Biswas and Kumar, 1989, 1990). For the small-r region it is sufficient to consider the radial equation as

$$R''_{nl}+\frac{2}{r}R'_{nl}+\left(\omega^2\varepsilon^2-\frac{l(l+1)}{r^2}\right)R_{nl}=0 \tag{4.27}$$

to demonstrate the bound-state structure. Approximating $\varepsilon^2 \simeq 1 - \frac{4}{3}\lambda_f r^2$ and putting

$$R_{nl} = (1/r)\,U_{nl}$$
$$\lambda^2 = -\tfrac{4}{3}\omega^2\lambda_f$$
$$\mu = \omega^2/2\lambda$$

we have that equation (4.27) reduces to

$$\frac{d^2U_{nl}}{dr^2} + \left[\omega^2 - \lambda^2 r^2 + \frac{l(l+1)}{r^2}\right] U_{nl} = 0$$

The solution is

$$U_{nl} = c_1 r^{l+1} \exp(-\lambda r^2)\; F_1(\tfrac{1}{2}(L + \tfrac{3}{2} - \mu),\, k + \tfrac{3}{2};\, \lambda r^2) \tag{4.28}$$

To ensure good asymptotic behavior, we impose the condition

$$\tfrac{1}{2}(l + \tfrac{3}{2} - \mu) = \eta \tag{4.29}$$

with $n = 0$, 1, 2, etc., which restricts the ω values to

$$\omega_{nl} = \left(\frac{16\lambda_f}{3}\right)^{1/2}\left(2n + l + \frac{3}{2}\right)$$
$$= B\left(M + \frac{1}{2}\right) \tag{4.30}$$

with $B = (16\lambda_f/3)^{1/2}$ and $M = 2n + l + 1$. For the electron, we introduce a potential term $V(\psi) = -\tilde{\gamma}^0 A\bar{\psi}\psi$ in the Lagrangian (4.5). Equation (4.30) is then modified to

$$E_{nl} = A + B(M + \tfrac{1}{2}) \tag{4.31}$$

In our model both e^+ and e^- act as gravitational objects bounded by a harmonic oscillator potential. So we get the energy of the pair e^+e^- as

$$E_P = 2A + 2B(M + \tfrac{1}{2}) \tag{4.32}$$

Table I gives the fitting corresponding to (4.32) for $A = 838$ keV and $B = 74$ keV.

Table I

n, l	M_{nl} (keV) equation (4.32)	M_{nl} (keV) (Cea, 1989)	$M_{\exp}(e^+e^-)$
(0, 0)	1471	1471	1471
(0, 1)	1646	1660	1646
(1, 0)	1794	1785	1782
(1, 1)	1942	1925	1837

In conclusion, the correlated narrow peaks observed in heavy-ion collisions are due to a time-dependent electromagnetic background creating the particles with consequent formation of a QED confining phase due to an anti-de Sitter-like static background.

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